Math 2FM3, Tutorial 3

Sep 27th , 2015

Annuity-Immediate

- Payments are made at the end of each period.

- Accumulated value:
- $s_{n|i} = (1+i)^{n-1} + \ldots + (1+i) + 1 = [(1+i)^n - 1]/i$
- Present value:
- $a_{n|i} = v + v^2 + \ldots + v^n = (1-v^n)/i$

Annuity-Due

- Payments are made at the beginning of the period.

- Accumulated Value:
- $\ddot{s}_{n|i} = (1+i)^n + \ldots + (1+i) = (1+i)\, s_{n|i} = [(1+i)^n - 1]/d$
- Present Value:
- $\ddot{a}_{n|i} = 1 + v + \ldots + v^{n-1} = (1 - v^n)/d$

Ex 2.1.1

- 50,000 can be invested under two options:

  Option 1

  Deposit the 50,000 into a fund earning an annual effective rate of i ; or

  Option 2

  Purchase an annuity-immediate with 24 level annual payments at an annual effective rate of 10%.

The payments under Option 2 are deposited into a fund earning an annual effective rate 5%. Both options produce the same accumulated value at the end of 24 years. Calculate i.

- Options 1: Accumulated value= $50{,}000(1+i)^{24}$

- Options 2: Annuity Payment is K,

  $K\, a_{24|\,0.1} = 50{,}000$, then K=5564.99

  $K\, s_{24|\,0.05} = 50{,}000(1+i)^{24}$, then i=6.9%.

Ex 2.2.1

- A 50,000 loan made on January 1, 2010 is to be repaid over 25 years with payments on the last day of each month, beginning January 31, 2010.

- a) If $i^{(2)} = 10\%$, find the amount of the monthly payment X.

- b) Starting with the first payment, the borrower decides to pay an additional 100 per month, on top of the regular payment of X, until the loan is repaid. An additional fractional payment might be necessary one month after the last regular payment of X+100. On what date will the final payment of X+100 be made, and what will be the amount of the additional fractional payment?

- (a) monthly rate $j=(1+i)^{1/12}-1$,

  i is annual rate and

  $1+i=[1+i^{(2)}/2]^2$, then $j=(1+0.1/2)^{1/6}-1$

  since X is monthly payment,

  $Xa_{300|j}=50{,}000$

  $X=447.24$

(b) $(X+100)a_{n|j}=50{,}000$, then $n=168.5$

168th payment occurs on Dec 31, 2023 by $X+100=547.24$

The fractional part $50{,}000-547.240a_{168|j}=Yv_j^{169}$

then $Y=290.30$.

Ex 2.2.11

- A loan of 1000 is repaid with 12 annual payments of 100 each starting one year after the loan is made. The effective annual interest rate is 3.5% for the first 4 years. Find the effective annual interest rate I for the final 8 years.

- $1000 = 100a_{4|0.035} + 100a_{8|i}\, v_{0.035}{}^{4}$
- $a_{8|i} = 7.26$
- i=2.208%.